Chapter 9

Testing the Difference Between Two Means, Two Proportions, and Two Variances
Chapter 9 Overview

Introduction

- 9-1 Testing the Difference Between Two Means: Using the z Test
- 9-2 Testing the Difference Between Two Means of Independent Samples: Using the t Test
- 9-3 Testing the Difference Between Two Means: Dependent Samples
- 9-4 Testing the Difference Between Proportions
- 9-5 Testing the Difference Between Two Variances
Chapter 9 Objectives

1. Test the difference between sample means, using the $z$ test.

2. Test the difference between two means for independent samples, using the $t$ test.

3. Test the difference between two means for dependent samples.

4. Test the difference between two proportions.

5. Test the difference between two variances or standard deviations.
9.1 Testing the Difference Between Two Means: Using the $z$ Test

Assumptions:

1. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.

2. The standard deviations of both populations must be known, and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.
Hypothesis Testing Situations in the Comparison of Means

(a) Difference is not significant

Do not reject $H_0$: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is not significant.
Hypothesis Testing Situations in the Comparison of Means

(b) Difference is significant

Reject $H_0: \mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is significant.
Testing the Difference Between Two Means: Large Samples

Formula for the z test for comparing two means from independent populations

\[ z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}}} \]
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Section 9-1
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Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is $88.42 and the average room rate in Phoenix is $80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $5.62 and $4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

**Step 1: State the hypotheses and identify the claim.**

$H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$ (claim)

**Step 2: Find the critical value.**

The critical value is $z = \pm 1.96$. 
Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is $88.42 and the average room rate in Phoenix is $80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $5.62 and $4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

Step 3: Compute the test value.

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$
Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is $88.42 and the average room rate in Phoenix is $80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $5.62 and $4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

Step 3: Compute the test value.

$$
z = \frac{(88.42 - 80.61) - (0)}{\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}} = 7.45
$$
Step 4: Make the decision.
Reject the null hypothesis at $\alpha = 0.05$, since $7.45 > 1.96$.

Step 5: Summarize the results.
There is enough evidence to support the claim that the means are not equal. Hence, there is a significant difference in the rates.
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Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-1
Example 9-2
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Example 9-2: College Sports Offerings

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume $\sigma_1$ and $\sigma_2 = 3.3$.

<table>
<thead>
<tr>
<th>Males</th>
<th></th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>11 11 8 15</td>
<td>6 8 11 13 8</td>
</tr>
<tr>
<td>6</td>
<td>14 8 12 18</td>
<td>7 5 13 14 6</td>
</tr>
<tr>
<td>6</td>
<td>9 5 6 9</td>
<td>6 5 5 7 6</td>
</tr>
<tr>
<td>6</td>
<td>9 18 7 6</td>
<td>10 7 6 5 6</td>
</tr>
<tr>
<td>15</td>
<td>6 11 5 5</td>
<td>16 10 7 8 5</td>
</tr>
<tr>
<td>9</td>
<td>9 5 5 8</td>
<td>7 5 5 6 5</td>
</tr>
<tr>
<td>8</td>
<td>9 6 11 6</td>
<td>9 18 13 7 10</td>
</tr>
<tr>
<td>9</td>
<td>5 11 5 8</td>
<td>7 8 5 7 6</td>
</tr>
<tr>
<td>7</td>
<td>7 5 10 7</td>
<td>11 4 6 8 7</td>
</tr>
<tr>
<td>10</td>
<td>7 10 8 11</td>
<td>14 12 5 8 5</td>
</tr>
</tbody>
</table>

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Example 9-2: College Sports Offerings

Step 1: State the hypotheses and identify the claim.

\[ H_0: \mu_1 = \mu_2 \] and \[ H_1: \mu_1 \neq \mu_2 \] (claim)

Step 2: Compute the test value.

Using a calculator, we find

For the males: \( \bar{X}_1 = 8.6 \) and \( \sigma_1 = 3.3 \)

For the females: \( \bar{X}_2 = 7.9 \) and \( \sigma_2 = 3.3 \)

Substitute in the formula.

\[
z = \frac{\left( \bar{X}_1 - \bar{X}_2 \right) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(8.6 - 7.9) - (0)}{\sqrt{\frac{3.3^2}{50} + \frac{3.3^2}{50}}} = 1.06
\]
Example 9-2: College Sports Offerings

Step 3: Find the P-value.
For $z = 1.06$, the area is 0.8554.
The $P$-value is $1.0000 - 0.8554 = 0.1446$.

Step 4: Make the decision.
Do not reject the null hypothesis.

Step 5: Summarize the results.
There is not enough evidence to support the claim that colleges offer more sports for males than they do for females.
Confidence Intervals for the Difference Between Two Means

Formula for the $z$ confidence interval for the difference between two means from independent populations

$$
\left( \bar{X}_1 - \bar{X}_2 \right) - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < (\mu_1 - \mu_2)
$$

$$
< \left( \bar{X}_1 - \bar{X}_2 \right) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}
$$

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Example 9-3: Confidence Intervals

Find the 95% confidence interval for the difference between the means for the data in Example 9–1.

\[
(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2
\]

\[
< (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}
\]

\[
(88.42 - 80.61) - 1.96 \sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}} < \mu_1 - \mu_2
\]

\[
< (88.42 - 80.61) + 1.96 \sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}
\]

\[
7.81 - 2.05 < \mu_1 - \mu_2 < 7.81 + 2.05
\]

\[
5.76 < \mu_1 - \mu_2 < 9.86
\]
9.2 Testing the Difference Between Two Means: Using the $t$ Test

Formula for the $t$ test for comparing two means from independent populations with unequal variances

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where the degrees of freedom are equal to the smaller of $n_1 - 1$ or $n_2 - 1$. 
Testing the Difference Between Two Means: Using the $t$ Test

- There is a different formula for the $t$ test for comparing two means from independent populations with equal variances. To determine whether two sample variances are equal, the researcher can use an $F$ test.

- Note, however, that not all statisticians are in agreement about using the $F$ test before using the $t$ test. Some believe that conducting the $F$ and $t$ tests at the same level of significance will change the overall level of significance of the $t$ test. Their reasons are beyond the scope of this textbook. Because of this, we will assume that $\sigma_1 \neq \sigma_2$ in this textbook.
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Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-2
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Example 9-4: Farm Sizes

The average size of a farm in Indiana County, Pennsylvania, is 191 acres. The average size of a farm in Greene County, Pennsylvania, is 199 acres. Assume the data were obtained from two samples with standard deviations of 38 and 12 acres, respectively, and sample sizes of 8 and 10, respectively. Can it be concluded at $\alpha = 0.05$ that the average size of the farms in the two counties is different? Assume the populations are normally distributed.

Step 1: State the hypotheses and identify the claim.

$H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$ (claim)
Example 9-4: Farm Sizes

Step 2: Find the critical values.
Since the test is two-tailed, \( a = 0.05 \), and the variances are unequal, the degrees of freedom are the smaller of \( n_1 - 1 \) or \( n_2 - 1 \). In this case, the degrees of freedom are \( 8 - 1 = 7 \). Hence, from Table F, the critical values are -2.365 and 2.365.

Step 3: Find the test value.

\[
t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(191 - 199) - (0)}{\sqrt{\frac{38^2}{8} + \frac{12^2}{10}}}
\]

\[= -0.57\]
Example 9-4: Farm Sizes

Step 4: Make the decision.

Do not reject the null hypothesis.

Step 5: Summarize the results.

There is not enough evidence to support the claim that the average size of the farms is different.
Confidence Intervals for the Difference Between Two Means

Formula for the $t$ confidence interval for the difference between two means from independent populations with unequal variances:

$$
(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < (\mu_1 - \mu_2) < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}
$$

d.f. smaller value of $n_1 - 1$ or $n_2 - 1$. 

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Section 9-2
Example 9-5
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Example 9-5: Confidence Intervals

Find the 95% confidence interval for the difference between the means for the data in Example 9–4.

\[
(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2
\]

\[
< (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}
\]

\[
(191 - 199) - 2.365 \sqrt{\frac{38^2}{8} + \frac{12^2}{10}} < \mu_1 - \mu_2
\]

\[
< (191 - 199) + 2.365 \sqrt{\frac{38^2}{8} + \frac{12^2}{10}}
\]

\[-41.0 < \mu_1 - \mu_2 < 25.0\]
9.3 Testing the Difference Between Two Means: Dependent Samples

When the values are dependent, do a $t$ test on the differences. Denote the differences with the symbol $D$, the mean of the population differences with $\mu_D$, and the sample standard deviation of the differences with $s_D$.

$$t = \frac{\overline{D} - \mu_D}{s_D / \sqrt{n}}$$

with d.f. = $n - 1$ and where

$$\overline{D} = \frac{\sum D}{n} \quad \text{and} \quad s_D = \sqrt{\frac{n \sum D^2 - (\sum D^2)}{n(n-1)}}$$

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Section 9-3
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Example 9-6: Vitamin for Strength

A physical education director claims by taking a special vitamin, a weight lifter can increase his strength. Eight athletes are selected and given a test of strength, using the standard bench press. After 2 weeks of regular training, supplemented with the vitamin, they are tested again. Test the effectiveness of the vitamin regimen at $\alpha = 0.05$. Each value in the data represents the maximum number of pounds the athlete can bench-press. Assume that the variable is approximately normally distributed.

<table>
<thead>
<tr>
<th>Athlete</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before ($X_1$)</td>
<td>210</td>
<td>230</td>
<td>182</td>
<td>205</td>
<td>262</td>
<td>253</td>
<td>219</td>
<td>216</td>
</tr>
<tr>
<td>After ($X_2$)</td>
<td>219</td>
<td>236</td>
<td>179</td>
<td>204</td>
<td>270</td>
<td>250</td>
<td>222</td>
<td>216</td>
</tr>
</tbody>
</table>
Example 9-6: Vitamin for Strength

<table>
<thead>
<tr>
<th>Athlete</th>
<th>Before ($X_1$)</th>
<th>After ($X_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>210</td>
<td>219</td>
</tr>
<tr>
<td></td>
<td>230</td>
<td>236</td>
</tr>
<tr>
<td></td>
<td>182</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>205</td>
<td>204</td>
</tr>
<tr>
<td></td>
<td>262</td>
<td>270</td>
</tr>
<tr>
<td></td>
<td>253</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>219</td>
<td>222</td>
</tr>
<tr>
<td></td>
<td>216</td>
<td>216</td>
</tr>
</tbody>
</table>

Step 1: State the hypotheses and identify the claim.  
$H_0$: $\mu_D = 0$ and $H_1$: $\mu_D < 0$ (claim)

Step 2: Find the critical value.  
The degrees of freedom are $n – 1 = 8 – 1 = 7$. The critical value for a left-tailed test with $\alpha = 0.05$ is $t = -1.895$.  

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Example 9-6: Vitamin for Strength

Step 3: Compute the test value.

<table>
<thead>
<tr>
<th>Before ($X_1$)</th>
<th>After ($X_2$)</th>
<th>$D = X_1 - X_2$</th>
<th>$D^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>219</td>
<td>-9</td>
<td>81</td>
</tr>
<tr>
<td>230</td>
<td>236</td>
<td>-6</td>
<td>36</td>
</tr>
<tr>
<td>182</td>
<td>179</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>205</td>
<td>204</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>262</td>
<td>270</td>
<td>-8</td>
<td>64</td>
</tr>
<tr>
<td>253</td>
<td>250</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>219</td>
<td>222</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>216</td>
<td>216</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$\Sigma D = -19 \quad \Sigma D^2 = 209$

$$\bar{D} = \frac{\sum D}{n} = \frac{-19}{8} = -2.375$$

$$s_D = \sqrt{\frac{n \sum D^2 - (\sum D)^2}{n(n-1)}} = \sqrt{\frac{8 \cdot 209 - (-19)^2}{8 \cdot 7}} = 4.84$$
Example 9-6: Vitamin for Strength

Step 3: Compute the test value.

\[ \bar{D} = -2.375, \quad s_D = 4.84 \]

\[ t = \frac{\bar{D} - \mu_D}{s_D / \sqrt{n}} = \frac{-2.375 - 0}{4.84 / \sqrt{8}} = -1.388 \]

Step 4: Make the decision. Do not reject the null.

Step 5: Summarize the results.

There is not enough evidence to support the claim that the vitamin increases the strength of weight lifters.
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Section 9-3
Example 9-7
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Example 9-7: Cholesterol Levels

A dietitian wishes to see if a person’s cholesterol level will change if the diet is supplemented by a certain mineral. Six subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per deciliter.) Can it be concluded that the cholesterol level has been changed at $\alpha = 0.10$? Assume the variable is approximately normally distributed.

<table>
<thead>
<tr>
<th>Subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before ($X_1$)</td>
<td>210</td>
<td>235</td>
<td>208</td>
<td>190</td>
<td>172</td>
<td>244</td>
</tr>
<tr>
<td>After ($X_2$)</td>
<td>190</td>
<td>170</td>
<td>210</td>
<td>188</td>
<td>173</td>
<td>228</td>
</tr>
</tbody>
</table>
Example 9-7: Cholesterol Levels

<table>
<thead>
<tr>
<th>Subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before ($X_1$)</td>
<td>210</td>
<td>235</td>
<td>208</td>
<td>190</td>
<td>172</td>
<td>244</td>
</tr>
<tr>
<td>After ($X_2$)</td>
<td>190</td>
<td>170</td>
<td>210</td>
<td>188</td>
<td>173</td>
<td>228</td>
</tr>
</tbody>
</table>

**Step 1: State the hypotheses and identify the claim.**

$H_0$: $\mu_D = 0$ and $H_1$: $\mu_D \neq 0$ (claim)

**Step 2: Find the critical value.**

The degrees of freedom are 5. At $\alpha = 0.10$, the critical values are $\pm 2.015$. 
Example 9-7: Cholesterol Levels

Step 3: Compute the test value.

<table>
<thead>
<tr>
<th>Before ($X_1$)</th>
<th>After ($X_2$)</th>
<th>$D = X_1 - X_2$</th>
<th>$D^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>190</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>235</td>
<td>170</td>
<td>65</td>
<td>4225</td>
</tr>
<tr>
<td>208</td>
<td>210</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>190</td>
<td>188</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>172</td>
<td>173</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>244</td>
<td>228</td>
<td>16</td>
<td>256</td>
</tr>
</tbody>
</table>

$\Sigma D = 100 \quad \Sigma D^2 = 4890$

$$\bar{D} = \frac{\sum D}{n} = \frac{100}{6} = 16.7$$

$$s_D = \sqrt{\frac{n\sum D^2 - (\sum D)^2}{n(n-1)}} = \sqrt{\frac{6 \cdot 4890 - (100)^2}{6 \cdot 5}} = 25.4$$
Example 9-7: Cholesterol Levels

Step 3: Compute the test value.

\[ \bar{D} = 16.7, \ s_D = 25.4 \]

\[ t = \frac{\bar{D} - \mu_D}{s_D / \sqrt{n}} = \frac{16.7 - 0}{25.4 / \sqrt{6}} = 1.610 \]

Step 4: Make the decision. Do not reject the null.

Step 5: Summarize the results.

There is not enough evidence to support the claim that the mineral changes a person’s cholesterol level.
Confidence Interval for the Mean Difference

Formula for the $t$ confidence interval for the mean difference

$$\bar{D} - t_{\alpha/2} \frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\alpha/2} \frac{s_D}{\sqrt{n}}$$

d.f. = $n = 1$
Example 9-8: Confidence Intervals

Find the 90% confidence interval for the difference between the means for the data in Example 9–7.

\[
\bar{D} - t_{\alpha/2} \frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\alpha/2} \frac{s_D}{\sqrt{n}}
\]

\[
16.7 - 2.015 \cdot \frac{25.4}{\sqrt{6}} < \mu_D < 16.7 + 2.015 \cdot \frac{25.4}{\sqrt{6}}
\]

\[
16.7 - 20.89 < \mu_D < 16.7 + 20.89
\]

\[-4.19 < \mu_D < 37.59
\]

Since 0 is contained in the interval, the decision is to not reject the null hypothesis \( H_0: \mu_D = 0 \).
9.4 Testing the Difference Between Proportions

\[
z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}
\]

where

\[
\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} \quad \hat{p}_1 = \frac{X_1}{n_1} \quad \hat{p}_2 = \frac{X_2}{n_2}
\]

\[
\overline{p} = 1 - \hat{p}
\]

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Section 9-4
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Example 9-9: Vaccination Rates

In the nursing home study mentioned in the chapter-opening Statistics Today, the researchers found that 12 out of 34 small nursing homes had a resident vaccination rate of less than 80%, while 17 out of 24 large nursing homes had a vaccination rate of less than 80%. At $\alpha = 0.05$, test the claim that there is no difference in the proportions of the small and large nursing homes with a resident vaccination rate of less than 80%.

$\hat{p}_1 = \frac{X_1}{n_1} = \frac{12}{34} = 0.35$ \hspace{1cm} and \hspace{1cm} $\hat{p}_2 = \frac{X_2}{n_2} = \frac{17}{24} = 0.71$

$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{12 + 17}{34 + 24} = \frac{29}{58} = 0.5, \quad \bar{q} = 0.5$
Example 9-9: Vaccination Rates

Step 1: State the hypotheses and identify the claim.

$H_0: p_1 - p_2 = 0$ (claim) and $H_1: p_1 - p_2 \neq 0$

Step 2: Find the critical value.

Since $\alpha = 0.05$, the critical values are -1.96 and 1.96.

Step 3: Compute the test value.

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{pq \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(0.35 - 0.71) - (0)}{\sqrt{(0.5)(0.5)\left( \frac{1}{34} + \frac{1}{24} \right)}}$$

$$= -2.7$$
Step 4: Make the decision.
Reject the null hypothesis.

Step 5: Summarize the results.
There is enough evidence to reject the claim that there is no difference in the proportions of small and large nursing homes with a resident vaccination rate of less than 80%.
Chapter 9
Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-4
Example 9-10
Page #506
Example 9-10: Missing Work

In a sample of 200 workers, 45% said that they missed work because of personal illness. Ten years ago in a sample of 200 workers, 35% said that they missed work because of personal illness. At $\alpha = 0.01$, is there a difference in the proportion?

To compute $\bar{p}$, you must find $X_1$ and $X_2$.

$$X_1 = \hat{p}_1 n_1 = 0.45 \times 200 = 90$$

$$X_2 = \hat{p}_2 n_2 = 0.35 \times 200 = 70$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{90 + 70}{200 + 200} = \frac{160}{400} = 0.4, \quad \bar{q} = 0.6$$
Example 9-10: Missing Work

Step 1: State the hypotheses and identify the claim.

\[ H_0: p_1 = p_2 \text{ and } H_1: p_1 \neq p_2 \text{ (claim)} \]

Step 2: Find the critical value.

Since \( \alpha = 0.01 \), the critical values are -2.58 and 2.58.

Step 3: Compute the test value.

\[
z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.45 - 0.35) - (0)}{\sqrt{(0.4)(0.6)\left(\frac{1}{200} + \frac{1}{200}\right)}} = 2.04
\]
Example 9-10: Missing Work

Step 4: Make the decision.
Do not reject the null hypothesis.

Step 5: Summarize the results.
There is not enough evidence to support the claim that there is a difference in proportions.
Confidence Interval for the Difference Between Proportions

Formula for the confidence interval for the difference between proportions

\[(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} < p_1 - p_2 \]

\[< (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \]
Chapter 9
Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-4
Example 9-11
Page #508
Example 9-11: Confidence Intervals

Find the 95% confidence interval for the difference of the proportions for the data in Example 9–9.

\[
\hat{p}_1 = \frac{X_1}{n_1} = \frac{12}{34} = 0.35 \quad \text{and} \quad \hat{q}_1 = 1 - 0.35 = 0.65
\]

\[
\hat{p}_2 = \frac{X_2}{n_2} = \frac{17}{24} = 0.71 \quad \text{and} \quad \hat{q}_2 = 1 - 0.71 = 0.29
\]

\[
\left(\hat{p}_1 - \hat{p}_2\right) - z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} < p_1 - p_2
\]

\[
< \left(\hat{p}_1 - \hat{p}_2\right) + z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}
\]
Example 9-11: Confidence Intervals

Find the 95% confidence interval for the difference of the proportions for the data in Example 9–9.

\[(0.35 - 0.71) - 1.96 \sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}} < p_1 - p_2\]

\[< (0.35 - 0.71) + 1.96 \sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}}\]

\[-0.36 - 0.242 < p_1 - p_2 < -0.36 + 0.242\]

\[-0.602 < p_1 - p_2 < -0.118\]

Since 0 is not contained in the interval, the decision is to reject the null hypothesis \(H_0: p_1 = p_2\).
In addition to comparing two means, statisticians are interested in comparing two variances or standard deviations.

For the comparison of two variances or standard deviations, an $F$ test is used.

The $F$ test should not be confused with the chi-square test, which compares a single sample variance to a specific population variance, as shown in Chapter 8.
Characteristics of the $F$ Distribution

1. The values of $F$ cannot be negative, because variances are always positive or zero.

2. The distribution is positively skewed.

3. The mean value of $F$ is approximately equal to 1.

4. The $F$ distribution is a family of curves based on the degrees of freedom of the variance of the numerator and the degrees of freedom of the variance of the denominator.

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Shapes of the $F$ Distribution
Testing the Difference Between Two Variances

\[ F = \frac{s_1^2}{s_2^2} \]

where the larger of the two variances is placed in the numerator regardless of the subscripts. (See note on page 518.)

The \( F \) test has two terms for the degrees of freedom: that of the numerator, \( n_1 - 1 \), and that of the denominator, \( n_2 - 1 \), where \( n_1 \) is the sample size from which the larger variance was obtained.
Chapter 9
Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-5
Example 9-12
Page #514
Example 9-12: Table H

Find the critical value for a right-tailed $F$ test when $\alpha = 0.05$, the degrees of freedom for the numerator (abbreviated d.f.N.) are 15, and the degrees of freedom for the denominator (d.f.D.) are 21.

Since this test is right-tailed with a 0.05, use the 0.05 table. The d.f.N. is listed across the top, and the d.f.D. is listed in the left column. The critical value is found where the row and column intersect in the table.
Example 9-12: Table H

Find the critical value for a right-tailed $F$ test when $\alpha = 0.05$, the degrees of freedom for the numerator (abbreviated d.f.N.) are 15, and the degrees of freedom for the denominator (d.f.D.) are 21.

$$\alpha = 0.05$$

<table>
<thead>
<tr>
<th>d.f.D.</th>
<th>1</th>
<th>2</th>
<th>$\cdots$</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vdots$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$F = 2.18$
Chapter 9
Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-5
Example 9-13
Page #514
Example 9-13: Table H

Find the critical value for a two-tailed $F$ test with $\alpha = 0.05$ when the sample size from which the variance for the numerator was obtained was 21 and the sample size from which the variance for the denominator was obtained was 12.

When you are conducting a two-tailed test, $\alpha$ is split; and only the right tail is used. The reason is that $F \geq 1$.

Since this is a two-tailed test with $\alpha = 0.05$, the $0.05/2 = 0.025$ table must be used.

Here, d.f.N. = $21 - 1 = 20$, and d.f.D. = $12 - 1 = 11$. 
Example 9-13: Table H

Find the critical value for a two-tailed $F$ test with $\alpha = 0.05$ when the sample size from which the variance for the numerator was obtained was 21 and the sample size from which the variance for the denominator was obtained was 12.

$\alpha = 0.025$

\[
\begin{array}{c|cccc}
\text{d.f.D.} & 1 & 2 & \cdots & 20 \\
\hline
1 & & & & \\
2 & & & & \\
\vdots & & & & \\
10 & & & & \\
11 & & & & \\
12 & & & & \\
\end{array}
\]

$F = 3.23$

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Notes for the Use of the $F$ Test

1. The larger variance should always be placed in the numerator of the formula regardless of the subscripts. (See note on page 518.)

2. For a two-tailed test, the $\alpha$ value must be divided by 2 and the critical value placed on the right side of the $F$ curve.

3. If the standard deviations instead of the variances are given in the problem, they must be squared for the formula for the $F$ test.

4. When the degrees of freedom cannot be found in Table H, the closest value on the smaller side should be used.
Assumptions for Using the $F$ Test

1. The populations from which the samples were obtained must be normally distributed. (Note: The test should not be used when the distributions depart from normality.)

2. The samples must be independent of each other.
Chapter 9
Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-5
Example 9-14
Page #516
Example 9-14: Heart Rates of Smokers

A medical researcher wishes to see whether the variance of the heart rates (in beats per minute) of smokers is different from the variance of heart rates of people who do not smoke. Two samples are selected, and the data are as shown. Using $\alpha = 0.05$, is there enough evidence to support the claim?

<table>
<thead>
<tr>
<th>Smokers</th>
<th>Nonsmokers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1 = 26$</td>
<td>$n_2 = 18$</td>
</tr>
<tr>
<td>$s_1^2 = 36$</td>
<td>$s_2^2 = 10$</td>
</tr>
</tbody>
</table>

Step 1: State the hypotheses and identify the claim.

$H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 \neq \sigma_2^2$ (claim)
Example 9-14: Heart Rates of Smokers

Smokers  | Nonsmokers
---------|-----------
\[ n_1 = 26 \] | \[ n_2 = 18 \]
\[ s_1^2 = 36 \] | \[ s_2^2 = 10 \]

**Step 2: Find the critical value.**
Use the 0.025 table in Table H since \( \alpha = 0.05 \) and this is a two-tailed test. Here, d.f.N. = 25, and d.f.D. = 17. The critical value is 2.56 (d.f.N. 24 was used).

**Step 3: Compute the test value.**

\[
F = \frac{s_1^2}{s_2^2} = \frac{36}{10} = 3.6
\]
Example 9-14: Heart Rates of Smokers

Step 4: Make the decision.
Reject the null hypothesis, since $3.6 > 2.56$.

Step 5: Summarize the results.
There is enough evidence to support the claim that the variance of the heart rates of smokers and nonsmokers is different.
Chapter 9
Testing the Difference Between Two Means, Two Proportions, and Two Variances

Section 9-5
Example 9-15
Page #516
Example 9-15: Doctor Waiting Times

The standard deviation of the average waiting time to see a doctor for non-lifethreatening problems in the emergency room at an urban hospital is 32 minutes. At a second hospital, the standard deviation is 28 minutes. If a sample of 16 patients was used in the first case and 18 in the second case, is there enough evidence to conclude that the standard deviation of the waiting times in the first hospital is greater than the standard deviation of the waiting times in the second hospital?

Step 1: State the hypotheses and identify the claim.

\[ H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{and} \quad H_1 : \sigma_1^2 > \sigma_2^2 \quad \text{(claim)} \]
Example 9-15: Doctor Waiting Times

The standard deviation of the average waiting time to see a doctor for non-lifethreatening problems in the emergency room at an urban hospital is 32 minutes. At a second hospital, the standard deviation is 28 minutes. If a sample of 16 patients was used in the first case and 18 in the second case, is there enough evidence to conclude that the standard deviation of the waiting times in the first hospital is greater than the standard deviation of the waiting times in the second hospital?

Step 2: Find the critical value.

Here, d.f.N. = 15, d.f.D. = 17, and $\alpha = 0.01$. The critical value is $F = 3.31$. 

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Example 9-15: Doctor Waiting Times

Step 3: Compute the test value.

\[ F = \frac{s_1^2}{s_2^2} = \frac{32^2}{28^2} = 1.31 \]

Step 4: Make the decision.
Do not reject the null hypothesis since 1.31 < 3.31.

Step 5: Summarize the results.
There is not enough evidence to support the claim that the standard deviation of the waiting times of the first hospital is greater than the standard deviation of the waiting times of the second hospital.